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SUMMARIES OF PAPERS PRESENTED AT THE MEETING ON THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR FUSION, HELD AT NEW YORK UNIVERSITY, APRIL 30 - MAY 1, 1962

July 31, 1962

AEC Research and Development Report

NEW YORK UNIVERSITY

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Magneto-Fluid Dynamics Division
Courant Institute of Mathematical Sciences
New York University

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Session 1 - Monday, 9:30 a.m.

Chairman - A. Simon

1. On Numerical Calculations of the Compression of a Plasma Cylinder

A. C. KOLB
Naval Research Laboratory

2. On the Containment for Infinite Times of Particles in a Mirror Machine

CLIFFORD S. GARDNER
New York University

Post has shown that a single particle, with velocity vector not in a certain escape cone, is trapped in a mirror machine. At least such a particle is adiabatically trapped: if ε (Larmor radius) tends to zero, then T (containment time) tends to infinity. This follows from Alfven's discovery that the magnetic moment $v_\perp^2/B$ is adiabatically invariant. Kruskal's "all orders" generalization shows that T becomes infinite faster than any negative powers of ε. Numerical computations and experiments have failed to indicate any upper limit on T when ε is reasonably small (neglecting collisional effects).

Prof. J. Moser of New York University has recently proved an important theorem in the mathematical theory of dynamical systems which is applicable to this problem. It follows that

for small enough Larmor radius, substantially all the single-particle trajectories which are adiabatically trapped are in fact rigorously trapped for all time.

3. The Boundary Layer in the Confinement of a One-Dimensional Plasma

W. PASKIEVICI, A. SESTERO, and H. WEITZNER
New York University

In this paper we describe attempts to obtain a simple configuration in which a plasma composed of both ions and electrons is contained by static electric and magnetic fields. We consider the simplest non-trivial situation in which the one-component fields depend on one space coordinate only and the particle acceleration vector has only the non-vanishing components. We assume the contained plasma to be at rest and in thermal equilibrium with equal ion and electron temperatures and with no fields present. For the larger portion of the paper we require that there are no trapped particles in the boundary layer between the plasma and the vacuum; that is, we demand that all particle orbits start and end in the contained plasma. We assume that the particles interact with each other only through the average field produced by all the particles, that is, we employ the Vlasov equation for both species.

We discuss the "charge neutral" approximation and then

give certain expansions useful in the numerical treatment of the approximation and the exact solution. The essential conclusion of a numerical computation of the charge neutral curve is that the boundary layer without trapped particles cannot be essentially charge neutral throughout the layer. From a numerical solution to the full problem we find that the only solution is physically unacceptable as it contains a runaway ion jet and large electric fields. We are thus forced to consider more complicated containment configurations and we present two simple schemes, one with different electron and ion temperatures, and the other with trapped particles. We offer these examples only to show that some kind of confinement is indeed possible; we do not propose them as physically interesting -- or attainable -- states.

4. Runaways in a "Lorentz" Plasma

MARTIN D. KRUSKAL and IRA B. BERNSTEIN

Princeton University

A spatially uniform gas of electrons colliding (Fokker-Planck) with infinitely massive fixed ions but not with each other is considered. The development and fate of runaway electrons is treated appropriately for the asymptotic limiting case of small imposed electric field $\underline{E}$. In the region in velocity space of the main body of the electron distribution

(for velocities of order of thermal velocity), the collisions make the distribution spherically symmetric on a finite time scale; this spherical distribution then satisfies a much slower diffusion equation ($\partial/\partial t \sim E^2$) with a boundary condition such that electrons ebb away by escaping to "infinite" velocity (running away). On a velocity scale of order $E^{-1/2}$ times thermal velocity (the scale of immediate competition between the electric acceleration and the collisions) these runaway particles reappear as if from a source at zero velocity, of which the strength (i.e. the runaway rate) varies slower than the natural time scale ($\partial/\partial t \sim E^{3/2}$) in the region, so that the distribution function is quasi-stationary; it may be determined by finding the unique solution once and for all of an equation of "two-way heat equation" type with definite boundary conditions. The far runaway region, of velocities so large that collisions are apparently negligible compared to acceleration, is also examined; the cumulative effect of the collisions turns out to be unbounded, however, the particles eventually running away with a very slowly (logarithmically) spreading normal distribution of velocities orthogonal to $\underline{E}$.

Theoretical Description of the Astron E-Layer

WARREN HECKROTTE and V. KELVIN NEIL

Lawrence Radiation Laboratory, Livermore

The E-layer of the Astron thermonuclear device is a cylindrical sheet of relativistic electrons. The electron density is to be such that the self field of the layer in its central region is greater than the imposed external field, thus providing closed lines of magnetic field in this central region. The injection system and the external magnetic field are to be controlled so that the radius of the layer is approximately uniform along its length and the canonical angular momentum of all the electrons is about the same. Making the assumptions that the E-layer is of zero-thickness, constant radius and finite length, the electrons relativistic with unique canonical angular momentum, and the injection rate very slow and the layer charge neutralized, the build-up of the layer in time, the magnetic field configuration, and the electron distribution in phase space have been calculated.

From these results we can conclude for our model, that the layer is stable against the bunching instability. The layer is stable with respect to the B_θ instability until the self-magnetic field is 20% greater than that necessary for field reversal, and perhaps somewhat beyond this point. The problem of providing the time-dependent external field necessary to insure the desired layer properties is defined but not solved at this time.

Session 2 - Monday, 2 p.m.

Chairman - Burton Fried

6. Trapping and Prolonged Confinement of an Energetic Deuterium Plasma in a Static Cusped Magnetic Field

J. OSHER

Los Alamos Scientific Laboratory

The results of recent experiments[1] to test trapping[2] of an energetic plasma from a fast coaxial plasma gun in a static cusped magnetic field will be described. The time history of neutron signals, produced primarily by deuterons escaping out along the cusped field lines and producing D-D reactions with deuterons on the chamber walls, is taken as the primary indication of plasma confinement. This signal is observed to include a substantial exponential portion with an average lifetime ($\tau_{1/e}$) varying roughly as the cube of the cusp magnetic field from 10 μsec at 2 kG up to 1 msec at 7.4 kG. The long $\tau_{1/e}$ observed even with background D_2 pressures of up to 5×10^{-5} mm of Hg appear to require "burnout" of neutral gas. These and other measurements are consistent with 5 to 10 keV average D^+ energy plasma being confined at an average initial density of $2 \times 10^{12}/cm^3$ in a volume of $\approx 10^4$ cm^3 with an electron temperature $\geq$ 1.5 keV. The confinement times observed

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[1] J. E. Osher, Phys. Rev. Letters 8, 305 (1962).

[2] J. L. Tuck, Phys. Rev. Letters 3, 313 (1959).

for energetic deuterons are much longer than one would calculate[3,4] for the large nonadiabatic ion orbits (Larmor radii typically 5 cm) expected in the Mark II cusped field (machine radius 12.5 cm). A preliminary model proposed to explain the long energetic ion confinement times is to consider the electrons as the fundamental particle trapped in an adiabatic (for them) cusped mirror field. These electrons form a negative potential well (measured $\approx$ 1 kV negative) which would reduce the more nonadiabatic ion loss rate through the cusps. The well depth, occurring primarily across a sheath $\lesssim$ 2 cm thickness, would then adjust itself until the modified ion loss rate equals the nonadiabatic electron loss rates from processes such as Coulomb collisions (i.e., $\tau_{ion} \approx \tau_{ee}$). Qualitative predictions for this confinement model will be described.

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[3] H. Grad, Atomic Energy Commission Report NYO-9496, 1961 (unpublished).

[4] E. P. Gray and I. M. Miller, Applied Physics Laboratory, Johns Hopkins University, Silver Springs, Maryland, Report CPR-010, 1961 (unpublished).

7. Ten Millisecond Containment of an Energetic Plasma in a Cusped Magnetic Field

DONALD C. HAGERMAN

Los Alamos Scientific Laboratory

A hydromagnetic gun pulse injects deuterium plasma into

a biconical cusped magnetic field[1-3] (axial cusp distance 160 cm). Neutrons (~10^7), mostly produced by energetic (~10 kev) ions reacting with deuterium on the walls, monitor the decay of the plasma. After initial fluctuations the neutron flux decays in a reproducible manner. Between 2 and ~40 milliseconds after injection the neutron signal decays linearly; the time to half neutron intensity varies linearly with the magnetic field strength and for 2000 gauss reaches 28 milliseconds. The linear decay does not fall completely to zero but changes to a more gradual decline. If the base pressure of the system is raised to 6×10^{-5} Torr the neutron signal is exponential with e-fold times of several milliseconds. These times are 10^3 greater than charge exchange times; hence burn-out of the background gas must have occurred. Present estimates of plasma density are of order 10^{11} ions/cc.

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[1] J. L. Tuck, Phys. Rev. Letters 3, 313 (1959).

[2] D. C. Hagerman, Salzburg Conference on Plasma Physics (1961).

[3] J. E. Osher, Phys. Rev. Letters (to be published in the April 15, 1962 issue).

8. Comments on the New Results in Plasma Confinement

J. L. TUCK

Los Alamos Scientific Laboratory

The enclosed preprint reports on the achievement of remarkably long (and apparently stable) confinement in a cusped geometry filled by trapping from an externally incident plasma jet (entropy trapping).

Previously, capture and confinement in picket fences has not been striking. The change seems to be the result of exceeding some threshold in the gun performance, presumably in $1/2\ \rho v^2$. In the light of these results with picket fence Mark IIB, given in the preprint, efforts were redoubled by Hagerman to get the same effect in the already existing larger picket fence, Mark III, which has a diameter of 1.5 meters with magnetic field 2000 gauss. Now this also has reached the new regime. There are some important differences in performance, however. The ratio Larmor radius/machine radius is about 1/5 in both cases. But Hagerman's confinement is much longer -- 10 milliseconds to half intensity -- and the decay is, after 2 milliseconds, linear. When the neutron intensity has fallen to ~0.1 of its value at 2 milliseconds, it then becomes exponential. The dependences of the confinement times τ in these differing regimes on magnetic field B are different, thus $\tau\ 1/e \propto B^3$ in IIB (200-2000 μsec), while $\tau_{1/2} \propto B$ in III (2000 μsec - 40 milliseconds).

One thing is worth noting about the 10 millisecond result; it seems beyond all question to require the achievement of burnour. Whereas for the IIB machine, one might sneak out of it by stretching the numbers.

It seems to me that these systems should snap back out of the burnt out state at some point in the decay when the neutral gas flux from the walls exceeds the burnout power of the decaying plasma. In this connection the time taken for the walls to return the system to 1/e of the original pressure is of interest. From the volume and pumping speed, it is about 5 sec. At this rate, the contribution of background gas from the gun is probably greater. A snap back effect has not yet been observed with certainty, though the change from linear to exponential is suggestive of some change in escape mechanism with density. It should be realized that the effect of a sudden increase in charge exchange is complicated -- for example, throwing particles out randomly instead of escaping through the cusps, changing the geometry of the neutron sources with respect to the detectors.

The β in these two machines are not known and can only be estimated roughly -- 10^{-2} in IIB and 10^{-3} in III. Hagerman has fairly clear indications in III that the plasma resides on the curved lines on the far side away from the gun (from the observation that no neutrons are emitted from the end walls on the inlet side of the mid-plane). Such a geometry seems undistinguishable in principle from that of a mirror machine albeit a distorted one. Now the puzzling thing is that the confinement

times we observe are much longer than those for mirror machines at comparable β. For, as found originally by Golovin and Joffe and now acknowledged pretty well universally, mirror machines at $\beta > 10^{-3}$ seem to be flute unstable. Incidentally the flute instability usually has the property of ending confinement suddenly. The gradually declining signal seen in these picket fence experiments is new and suggestive of stability. Can it be that the peculiar warped shape of the mirror machine existing in a low β picket fence imparts flute stability by some precessional effect?

It has been suggested by Osher that the excessively long confinement time is due to the presence of a confining electrostatic field, and indeed the interior of the picket fence is observed to be 1 kv negative. He points out that such a field is to be expected since the electrons are highly adiabatic; but having such large ion Larmor orbits (~1/5 radius of the machine) the ions are not. The remarks of Berkowitz et al. in the Geneva Conference reports are noteworthy on this subject.

One wonders how 1 kv small potential could have any effect on 5-20 kev ions. However,it may be that the ion motion at large radius in the ring cusp is mainly tangential so that only small energy is available for escape in the radial direction. A small potential could then turn such ions back.

It may be worth observing that though for a mirror machine a bundle of planar coaxial noncolliding orbits is possible (exactly like particles coasting in a cyclotron which latter

is after all a truncated mirror machine), no such simply periodic noncolliding orbits seem possible in the cusped system. Hence the long confinements observed are all the more interesting, in that they must involve complex three dimensional intersecting orbital motions.

On the subject of the relative contributions of wall reactions and volume thermonuclear reactions to the total observed neutro signal: The wall reaction yield Y_a is given by

$$Y_a = n\ V\ F(T)$$

(where F = adsorbed target yield per deuteron, T = temperature, V volume, τ confinement time, $\overline{\sigma V}$ Maxwell averaged cross section.

and the thermonuclear yield by

$$Y_b = \frac{1}{2}\ n^2 V \langle \overline{\sigma V} \rangle\ \tau$$

and the ratio $Y_b/Y_a \propto n\tau$. Calculations suggest that Y_b/Y_a should be of order of magnitude 1/10 in these experiments, though Y_b has not yet been detected experimentally. But as density and time are increased, so also must the proportion which is thermonuclear.

Which brings up the constantly recurring performance criterion - $n\tau$. Thus the Lawson criterion for self maintenance in DT is $n\tau = 10^{14}$ (T ~ 20 kev). In terms of $n\tau$, the new performance in picket fence is not outstanding, being comparable to those achieved in the DCX and Table Top machines but far below that in the Scylla family. Indication of an even higher $n\tau$ in a small fraction of a mm^3 in the dynamic pinch of Andrianov,

N. V. Phillipov, et al., were reported at Salzburg. Here the confinement is essentially inertial, and we are in a different regime of reactors (I have always called it the Columbus X regime), intermediate between bombs and bottles.

	$n(cm^{-3})$	τ(sec)	$n\tau$	T_i or E kev	Neutrons
DCX	10^{7}	37	4×10^{8}	~300 kev	---
Toy Top	10^{13}	1.4×10^{-4}	1.4×10^{9}	~3.5 kev	$\sim 10^{5}$
Picket Fence IIB	10^{12}	10^{-3}	10^{9}	~5-10 kev	$\sim 10^{6}$
III	10^{11}	10^{-2}	10^{9}	~5-10 kev	$\sim 10^{6}$
Scylla III	10^{17}	2×10^{-6}	2×10^{11}	~1.9 kev	5.10^{8}
Andrianov, N.V. Phillipov, et al	10^{19}	2×10^{-7}	2×10^{12}	~1 kev	5×10^{9}

Another way of rating confinement performance, which amounts to the same thing but which is physically more illuminating, is to express it in terms of ion-ion collision times, temperature being assumed adequate. For a thermonuclear reactor, confinement must be for 100 to 1000 ion-ion collision times. (Non-thermonuclear devices may not need so many.)

In my opinion, such criteria as these have been unduly neglected in setting out achievements in plasma confinement. The adiabatic compression family of devices (Scylla, Pharos, the GE, the UK, the Aachen devices, and Caride) have in some cases held a plasma together for $\geq 90^{\circ}$ ion collisions per ion -- at a

temperature of ~2 kev. This is not quite hot enough for a reactor, or anywhere near long enough, but moving in the right direction. Expressed in these terms, the confinements in the picket fence and any other bottles are an order of magnitude less. Since a hydromagnetic instability can end confinement in less than a collision time, observations of long time collisionless confinement are encouraging to the extent that the confinement is better than it might have been.

Which brings us to the concluding point. The plasma temperature in these picket fences is probably already sufficient for reactor purposes, though barely so. It is probably over-optimistic to expect that these newly observed confinement times in picket fences and their B dependence will continue on to higher densities and fields. But if it should, then the energy breakeven point for a picket fence thermonuclear reactor turns out to be in the region of $n \sim 10^{15}$, $\tau \sim 1/4$ sec, $T \sim 10$ kev, and $B \sim 50$ kilogauss.

9. On The Theoretical Interpretation of Cusped Containment Experiments

HAROLD GRAD

New York University

The estimation of containment in a Picket Fence or Cusped Geometry is complicated by many factors. There are adiabatic and nonadiabatic parts of both the ion and electron population,

each with its own containment mechanism and nominal containment time. These times differ by many orders of magnitude and must be balanced against one another to obtain the actual containment time.[1] These individual loss rates are strongly affected by electric fields perpendicular to B on which depends the type of adiabatic invariant which governs reflection at a mirror (in other words, the effective "hole" size).[2] To be effective, this potential does not have to be comparable to kT in magnitude. A different effect is the equalization of nominal electron and ion losses which is effected by an electric field along B; as a rough rule, the net loss rate is the smaller of the two. Scaling of the adiabatic and nonadiabatic component losses (which combine to give the total nominal electron or ion rate) is radically different with respect to n, T, B, as well as shape and size.[3]

Applying this theory to the Osher and Hagerman experiments indicates that electrons are almost entirely adiabatic and ions mostly nonadiabatic. The observed thin sheath (perpendicular component of E) and corresponding thin "hole" size (which was predicted as a necessary requirement for good containment in

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1 For a general survey see H. Grad, "Containment in Cusped Plasma Systems", NYO-9496, March 1961.

2 H. Grad, "General Adiabatic Theory", TID-7520, p. 402, Sept. 1956.

3 See H. Grad, "Developments in the Theory of Cusped Geometries", TID-7520, p.148, Sept. 1956; more recently, "Thermonuclear Plasma Containment in Open-Ended Systems, NYO-9355, Sept.1960.

the thermonuclear range[4]) indicates a nonadiabatic ion containment time which, although 5-10 times higher than the uncorrected value (ignoring the sheath potential), is still much shorter than the nominal electron time. On the other hand, any adiabatic ions will have a containment time which is much longer than the nominal electron time. Therefore the net ion time may be extremely sensitive to the presence of a small number of adiabatic ions, and the range of possibilities surrounds the nominal electron time. Since the shorter of the two times, net electron and ion, is the controlling one, the observed containment will be sensitive to a change in the net ion time only if this is the shorter of the two; otherwise it will depend almost entirely on electron containment. Either the nominal electron time or a time somewhere between the extremes of the two kinds of ions is not inconsistent with the observed decay times. However, the unusual scaling as B^3 in Osher's experiment (which is inconsistent with either purely adiabatic or purely nonadiabatic containment) as well as the complicated transitions observed with time indicate that the experiment lies in a transition regime (transferring from nonadiabatic to adiabatic as B increases) or else, as suggested by Osher, the scaling may be due to "gun" effects which lead to variations in the state of the injected electron plasma.

In any event, one cannot conclude, at this time, that there

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[4] H. Grad, "Theory of the Cuspidor", TID-7503, p.319, Feb. 1956.

is any discrepancy between theory and experiment. The most likely method of reducing the ambiguity of the comparison between theory and experiment would seem to be to experimentally determine the split between adiabatic and nonadiabatic components, e.g., by varying the relative field strengths at the two mirrors.[5]

While the observed containment time is very interesting in itself, the ultimate significance is very hard to predict because of the transitional nature of the present experiments with regard to scaling. To become interesting for thermonuclear purposes, one must ultimately reach a regime in which the net ion containment is much better than adiabatic electron confinement, and the present experiments can only begin to shed light on this region.

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[5] See footnote 1.

10. R. F. Enhancement of Single Particle Confinement in Cusped Geometries*

ERNEST P. GRAY

Applied Physics Laboratory, The Johns Hopkins University

The possibility of plugging the ring and point cusps of a magnetic monocusp by appropriate r.f. fields has been studied. By a numerical integration of single-particle

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* This work was supported by the Bureau of Naval Weapons, Department of the Navy.

trajectories, the critical r.f. field strength for particle reflection at the cusp -- defined as that field strength which causes the particle to be reflected just at the cusp -- has been obtained as a function of the various parameters, e.g., frequency, static magnetic field strength, particle energy, initial position and initial heading. It was found that greatly enhanced confinement can be achieved at a relatively modest r.f. power level by using r.f. fields with the electric field direction more or less perpendicular to the magnetic field lines of the cusp to be plugged, and at such a frequency that the particle to be trapped will experience cyclotron resonance on its way to the cusp. As a typical example, a proton of initial energy 1 Kev, starting at the center of a monocusp 20 cm in radius and 20 cm in length, with the field rising to 30 kilogauss at both ring and point cusps, cannot escape in a single transit, if an appropriate 30-megacycle field is applied with a maximum electric field strength of about 1200 volts/cm. If a Q of 100 can be attained, no more than 75 kilowatts of r.f. power are required, corresponding to a power dissipation density of 10 watts/cm^3 (or about 10% of the desired fusion power generation density). Although this represents a sizable power dissipation density, it need be applied only over a relatively small volume near the cusps. The enhanced confinement so obtained is not perfect, however, because the particle eventually gains too much energy to remain confined. After some 10 to 50 transits (the exact number

depending sensitively on the r.f. field strength and frequency), the particle escapes at an energy of some 20 to 40 times its initial value.

11. Some Problems of Nonadiabatic Plasma Dynamics

GEORGE SCHMIDT

Stevens Institute of Technology

Particles in adiabatic motion are attached to flux surfaces in fields of axial symmetry. It is shown that in the case of "fast" changes in the field (as measured in the particle coordinate system), the particle tends to stick to its flux surface, even at the price of sacrificing the constancy of μ_m. There are circumstances, however, when this is not possible. If, e.g., a quasi-uniform magnetic field is reversed in time (second half cycle in a θ-pinch) the first half cycle flux surfaces are nowhere to be found in the second half cycle. The same is true for a particle passing through the median plane of a cusp geometry. In these cases the particle motion exhibits various peculiarities. Its angular momentum, e.g., $mv_\theta r = P_\theta - qrA_\theta$ is strongly asymmetrical for the change of sign of A_θ, (note that P_θ = const.). The lower bound of the energy picked up by a particle in the second half cycle of a θ-pinch is calculated and found to be very large for electrons (several MeV).

With the breakdown of adiabaticity for single particles the

basis for hydromagnetic treatment also breaks down for a collisionless plasma. A simple self-consistent field reversal problem is investigated. At t = 0 a cylindrical shell of cold particles is located in a uniform magnetic field of $(P_\theta) = n\delta(P_\theta - P_\theta^o)$. After the (slow) reversal of the external field the shell is found in fast gyration, setting up currents weakening or even reversing the external field inside. In short, an E layer is automatically established.

Experiments performed and planned are also shortly discussed.

Session 3 - Tuesday, 9:30 a.m.

Chairman - Warren Heckrotte

12. Instabilities in Inhomogeneous Plasmas

E. A. FRIEMAN and J. M. GREENE

Princeton University

We will give a progress report on our work on the stability of inhomogeneous plasmas. Our equilibrium consists of a magnetic field in the z-direction supporting a plasma whose pressure is comparable to B^2, and whose temperature and density vary in the x-direction. The Larmor orbit diameter of the particles is taken to be much smaller than the scale length of the density and temperature variation. We have been considering wave propagation perpendicular to the magnetic field. We have shown that the magnetoacoustic wave is stable and have identified an unstable mode with a transverse electric field. The latter mode was previously reported by Krall and Rosenbluth.

13. Crossed-Stream Instability

E. G. HARRIS

University of Tennessee and Oak Ridge National Laboratory

We consider a beam of energetic ions moving perpendicular to a magnetic field through a background plasma. We show that an instability exists which is similar to the familiar two-

stream instability. If differs from it in that the electron motion is essentially parallel to the magnetic field and, hence, perpendicular to the direction of ion streaming. When the ion paths close on themselves and the electrons are cold, we recover previous results.[1] Our present work includes the effect of electron temperature. A nonzero electron temperature increases the instability.

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[1] P. Burt and E.G. Harris, Phys. Fluids 4, 1412 (1961).

14. Instability of a Low Pressure Plasma Column in a Longitudinal Magnetic Field*

N. A. KRALL and A. SIMON

General Atomic

The stability of a weakly ionized, low density plasma in a uniform magnetic field was considered in a previous publication.[1] The plasma is a finite conducting chamber; there is diffusion across the field lines and streaming to the end walls. An unstable helical mode results for magnetic fields above some critical value.

The expression for the critical field value obtained above was based on the assumption that the eigenfunction could be

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* This work was done under a joint General Atomic-Texas Atomic Energy Research Foundation program on controlled thermonuclear reactions.

[1] G. Guest and A. Simon, Phys. Fluids 5, 503 (1962). See also GA-2671.

chosen to be entirely real. We have now investigated the validity of this approximation by direct numerical solution of the complex eigenvalue equations. Good agreement between the numerical eigenvalues and the analytic theory resulted. The variation of the eigenvalues with the parameters of the problem was in accord with the analytic theory. The nature of the actual eigenfunctions will be discussed.

In addition to the numerical check, one can solve the complex equations exactly in the limit of a small parameter (which is proportional to the magnetic field). This exact result agreed very well in this limit with both the numerical results and the approximate theory yielding the real and imaginary part of the eigenfrequency ω as well as the eigenfunctions.

15. Dispersion Relation for Fokker-Planck Equation and the Effect of Collisions on Ion Wave Instability

C. S. SHEN and R. M. KULSRUD

Princeton University

The dispersion relation for electrostatic oscillations in a small external electric field is derived on the basis of the Fokker-Planck equation. The effect of the external field and collisions on the growth rate of ion wave instability is discussed under the following assumptions: (1) the influence of external field and collisions on the perturbed distribution

is small; (2) the undisturbed distribution functions of ions and electrons are Maxwellian and shifted Maxwellian respectively; (3) the thermal energy of ions is small compared to that of electrons. The critical current for the onset of instability is obtained by taking collisions and external fields into consideration. It is found that the external field tends to increase the growth rate of instability, but the collisions tend to damp it.

16. Secondary Plasma Growth

T. K. FOWLER

Oak Ridge National Laboratory

It is pointed out that, in a plasma containing electrons sufficiently energetic to ionize neutrals, the cold ion density increases abruptly above a critical neutral pressure such that the ionization time is less than the time for cold ions to drift out of the plasma. The ion density grows until the accompanying hot electrons reduce the ionization time by cleaning out the neutral population, a "secondary" burnout. This phenomenon appears to occur in DCX-1 and possibly accounts for a puzzling discontinuous change in density of a plasma formed by electron cyclotron heating when the pressure exceeds a critical value. In the latter case, the plasma potential would be predicted to change sign when the density jump occurs.

Session 4 - Tuesday, 2 p.m.

Chairman - Edward Harris

17. Quantum Mechanical Kinetic Equation of a Plasma

BURTON D. FRIED and H. W. WYLD, Jr.

Thompson Ramo Wooldridge Inc.
Canoga Park, California

The method of deriving kinetic equations for a classical plasma developed by Rosenbluth, Rostoker, Lenard et al., is generalized to the case of a plasma so dense that the effects of the uncertainty principle and the Pauli principle become significant. In place of the BGBYK hierarchy of equations for the distribution functions, f_s, one has a similar coupled set of equations for Wigner distribution functions, or (more convienently it turns out) for the Fourier transforms of these. Since, in general, there is no small expansion parameter analogous to the $n\lambda_D^3$ of the classical case, neglecting the three particle correlations has not the same justification as in the classical case. Nonetheless, doing so leads to a pair of coupled equations from which one can deduce most of the information and results of other fashionable approximation methods.

18. Kinetic Theory of an Electron-Photon Gas

H. DREICER

Los Alamos Scientific Laboratory

A Boltzmann-like transport equation for the distribution of electron moments is derived for an electron-photon gas in which electrons emit, and absorb photons independently. Under equilibrium conditions the equation is satisfied by a Maxwellian distribution for the particles provided the frequency distribution of the photons follows the Planck law at the same temperature. Application is made to the case of negative absorption of cyclotron radiation by the electrons of a collisionless plasma situated in a magnetic field. Negative absorption (or maser action) can occur when the electron velocity distribution possesses an appropriate positive slope, and provided transit time broadening of the cyclotron radiation is sufficiently small. Under these conditions the electrons radiate more energy by induced emission than they absorb by true absorption, and in the process their velocity distribution relaxes to a state closer to the equilibrium Maxwellian distribution. This relaxation process has been followed with the help of the Boltzmann-like transport equation described above by assuming either a Gaussian or a Lorentz frequency distribution for the amplified radiation field. The result indicates that this relaxation process gives rise to a broadening of the amplified line. The rate at which the velocity distribution relaxes in

the neighborhood of its maximum positive slope is proportional to the intensity of the radiated line, and inversely proportional to the cube of the line width of the radiation, i.e., $(\Delta\omega)^3$.

19. Thermalization of Fast Ion Groups in a Plasma*

HERBERT C. KRANZER

Adelphi College

The process of thermalization of a single fast ion injected into a plasma in equilibrium was studied in (1). The procedure used was numerical integration of the fully linearized Fokker-Planck equation with time-independent coefficients corresponding to an undisturbed plasma. In some proposed thermonuclear reactors, however, a whole group of ions would continuously transfer energy to the plasma during thermalization, significantly altering the coefficients in the Fokker-Plance equation. Accordingly, the computations of (1) have been repeated with two modifications. First, the initial ion distribution is taken as Maxwellian rather than as a delta function. Second, the plasma ions and electrons are allowed (separately) to increase in temperature as the injected ions cool down; the

* Research supported by the Atomic Energy Commission and the Office of Naval Research.

1 Kranzer, H. C., "Thermalization of a Fast Ion in a Plasma", Physics of Fluids 4, 214-220 (1961).

total energy of the system remains constant. In general, the mean time of thermalization for a group of fast ions is found to be longer than for a single ion at the same temperature.

20. Thermal Relaxation in a One-Species, One-Dimensional Plasma

JOHN DAWSON

Princeton University

The theories of Lenard[1], Rostoker and Rosenbluth[2], and Balescu[3] predict that stable non-Maxwell distributions do not relax to a Maxwell distribution for a one-species, one-dimensional plasma. These theories consider only the two-particle correlation function and apply only in the limit of a large n_d; n_d is the number per Debye length. This prediction has been verified by numerical experiments on a one-dimensional plasma model. Systems containing 1000 to 3000 particles were started with square velocity distributions. The number of particles in a Debye length ranged from 2.5 to 20. It was found that the time for relaxation to Maxwellian went as n_d^2.

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[1] A. Lenard, Ann. Phys. 10, 390 (1960).

[2] N. Rostoker and M. N. Rosenbluth, Phys. Fluids 3, 1 (1960).

[3] R. Balescu, Phys. Fluids 3, 52 (1960).

This relaxation is much slower than the relaxation of a test particle to the background distribution which goes as n_d.[4] A transient relaxation also showed up which required a time of the order of $1/\omega_p$, and which pulled the corners of the distribution in a distance $\Delta V = (2\pi\sigma^2 D/m)^{1/2}$ where σ is the charge on a sheet and D is the Debye length. The distribution also develops a tail which extended the distance ΔV beyond its initial edge. It was also found that though the mean distribution function remained square for quite a long time, there were rapid fluctuations about this distribution and the characteristic time for these was of the order of $1/\omega_p$.

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[4] J. Dawson, Phys. Fluids 5,445 (1962).

21. Irreversible Processes in a One-Dimensional Plasma*

O. ELDRIDGE and M. FEIX+

General Atomic

We investigate the test particle problem and the evolution of a non-Maxwellian velocity distribution, using a one-dimensional model of a plasma which allows us to calculate with

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* This work was done under a joint General Atomic-Texas Atomic Energy Research Foundation program on controlled thermonuclear reactions.

+ On leave of absence from Association Euratom CEA, Fontenay aux Roses.

an electronic computer the exact trajectories of particles. We obtain from the usual asymptotic theory the Fokker-Planck coefficients for a set of test particles, i.e., particles which are singled out from an equilibrium distribution. The numerical "experiments" show that the relaxation time for test particles varies as nD, the number of particles in a Debye length, as predicted by the theory. The magnitude of the coefficients also agrees with theory for $4 < nD < 20$. The first order theory predicts that the detailed balance between drag and diffusion is valid for all stable distribution, not only at equilibrium. So any change in the velocity distribution should be at least a second order effect. We study the evolution of two non-Maxwellian distributions, using the entropy as a measure of the deviation from a Maxwellian. The "experiments" show a large entropy change during the first half plasma period for all values of nD. The relaxation then goes at a much slower rate.

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